TABLE II. Energies and classification of recombination emission bands from diamond.[a]

Intrinsic		Extrinsic	
Band and threshold energy (eV)	Classification	Band and peak energy (eV)	Classification
A^b; 5.325±0.002	Type α, $E_g-E_x-\hbar\omega_1$ (5.325 eV)	C', 5.375±0.002	?
$B_1{}^b$; 5.276±0.001	Type α, $E_g-E_x-\hbar\omega_2$ (5.276 eV)	D', 5.361±0.002	?
$B_2{}^b$; 5.268±0.001	Type α, $E_g-E_x-\hbar\omega_3$ (5.268 eV)	X, 5.267±0.001	Type γ, $E_g-E_x-\hbar\omega_2$ (5.267 eV)$-E_{d_2}$
$B_3{}^b$; 5.245±0.002	Type β, $E_g-E_x-\hbar\omega_3$ (5.245 eV)$-\hbar\omega_4$	Y, 5.259±0.001	Type γ, $E_g-E_x-\hbar\omega_3$ (5.259 eV)$-E_{d_2}$
E^b; 5.133±0.003	Type β, $E_g-E_x-\hbar\omega_3$ (5.134 eV)$-\hbar\omega_2$	C, 5.222±0.001	Type γ, $E_g-E_x-\hbar\omega_2$ (5.220 eV)$-E_{d_1}$
F^b; 5.098±0.002	Type α? $E_g-E_x-\hbar\omega_3$ (5.103 eV)$-\hbar\omega_R$	D, 5.210±0.001	Type γ, $E_g-E_x-\hbar\omega_3$ (5.212 eV)$-E_{d_1}$
K^c; 4.987±0.003	Type α? $E_g-E_x-\hbar\omega_1$ (5.495 eV)$-2\hbar\omega_R$	G, 5.090±0.003	?
L^c; 4.922±0.006	Type α? $E_g-E_x-\hbar\omega_3$ (4.938 eV)$-2\hbar\omega_R$	H, 5.076±0.002	Type γ? $E_g-E_x-\hbar\omega_2$ (5.078 eV)$-\hbar\omega_3-E_{d_1}$
		I, 5.059±0.002	?
		J, 5.034±0.003	?

[a] Classification types: α—intrinsic exciton annihilation with phonon emission; β—intrinsic exciton annihilation involving intervalley scattering; γ—extrinsic exciton annihilation occurring at defect or impurity. Symbols: E_g—indirect energy gap; $\hbar\omega_i$—phonon energy—see text; E_x—exciton binding energy; $\hbar\omega_R$—Raman energy; E_{d_1}—binding energy of exciton to C, D defect-substitutional aluminum (magnitude 0.056 eV); E_{d_2}—binding energy of exciton to X, Y defect-unknown nature (magnitude 0.009 eV).
[b] Threshold energies of these bands obtained from extrapolation of fitted Maxwell-Boltzmann curves.
[c] Threshold energies of these bands obtained assuming difference between apparent threshold and M.B. fit is the same as for band F.

CONCLUSION

A comprehensive list of the emission bands is given in Table II together with a review of the interpretation scheme proposed above. The most serious shortcomings in the present scheme are encountered in dealing with the extrinsic bands (bands near to E_g-E_x). As previously noted, some of the band with lower values of E_i which are listed as intrinsic in Table II may have been enhanced by the effects of suitable defects.

ACKNOWLEDGMENTS

One of us (I.H.J.) is grateful to the Department of Industrial and Scientific Research for the provision of a maintenance grant during the course of this work. Thanks are due to A. Benfield, H. Lemke, and D. Risley for extensive technical assistance and advice. We are indebted to Dr. J. R. Haynes for communicating some unpublished results on silicon and for a stimulating discussion of this work.

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Piezoresistive Properties of Heavily Doped n-Type Silicon*

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The piezoresistance effect has been studied in n-type silicon over the impurity concentration range from approximately 1×10^{15} to 1×10^{20} cm^{-3}. From the analysis of the results, it is concluded that for Fermi energies up to 0.08 eV above the band edge, the total number of states below the Fermi level is correctly given by assuming that the band is parabolic with a density-of-states effective mass equal to that found in pure silicon. The qualitative dependence of the mobility anisotropy on impurity concentration and temperature is determined from the piezoresistance results. From the temperature dependence of the coefficient π_{11}, it is concluded that the deformation potential in n-type silicon increases by approximately 25% between 77 and 300°K. Finally, the π_{44} coefficient is found to increase with increasing impurity concentration at the highest concentrations. This behavior is attributed to the Fermi level approaching the energy of the conduction band at the [100] zone boundary in silicon having an electron concentration of the order of 1×10^{20} cm^{-3}.

I. INTRODUCTION

DURING the past decade, the properties of relatively pure n-type silicon have been extensively investigated to the point that it is today one of the best understood semiconductor materials. Cyclotron resonance, optical and galvanomagnetic measurements have clearly defined the energy-band structure near the band edge and have led to a quantitative understanding of the scattering mechanisms in lightly doped silicon. However, much less attention has been given to the properties of heavily doped silicon, where the term heavily doped here refers to doping levels between the concentration at which the impurity activation energy

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goes to zero and the solubility limit of the impurity. Consequently, the effects of heavy doping on the known properties of lightly doped silicon are not well understood.

The mobility of electrons in heavily doped silicon was first studied by Pearson and Bardeen.[1] Since then, the variation in the mobility with temperature and impurity concentration has been studied by several investigators[2–6] until at the present time the experimental behavior is quite well defined, but theoretically the mechanisms that limit the mobility in the heavily doped case are not understood.[6,7] This is especially true at the lower temperatures.

The effective mass of electrons in heavily doped silicon has been measured by Spitzer and Fan[8] using infrared reflectivity techniques. They found the conductivity effective mass to be unchanged at an impurity concentration of 3.6×10^{18} cm^{-3}. Later, Cardona *et al.*,[9] using the same techniques, concluded that the effective mass in heavily doped silicon is approximately 50% greater than in lightly doped silicon. Very recently, Howarth and Gilbert,[10] using the same techniques, conclude that the effective mass is unchanged in heavily doped silicon. Magnetic susceptibility measurements[11,12] indicate that the effective mass is unchanged for impurity concentrations up to approximately 5×10^{18} cm^{-3} but increases by approximately 25% at a concentration of 2.9×10^{19} cm^{-3}.[12] From specific-heat measurements, Keesom and Seidel[13] conclude that the density-of-states effective mass in silicon doped to 1×10^{19} cm^{-3} is 27% larger than in pure silicon. Thus, the situation as to the variation in the effective mass and changes in the shape of the conduction band due to heavy doping is not clear at the present time. However, it should be noted that the most recent results[10] indicate the effective mass is unchanged by heavy doping.

Recently, the effect of heavy doping on the shape of energy bands in semiconductors has been treated theoretically by several investigators.[14–18] The principal effect is found to be a rigid displacement in energy of the lightly doped band and a tailing of the density of states near the band edge.[15,17]

Tunneling studies[19] in degenerately doped silicon *p-n* junctions indicate the presence of band tails. However, the results of tunneling experiments are not sufficiently quantitative to give useful information about the shape of the bands away from the band edge. It is therefore useful to use electrical transport effects in heavily doped silicon to study the shape of the energy bands at points away from the band edge.

The piezoresistance effect is a very useful tool for studying the band structure in heavily doped semiconductors as shown by Pollak[20] in his work on *n*-type germanium. In *n*-type silicon, the piezoresistance effect will also be sensitive to the mobility anisotropy, so that the piezoresistance can be used to study both the mobility anisotropy and energy-band structure. In an earlier paper,[21] the piezoresistance effect was investigated in silicon-diffused layers and the results indicated certain anomalies in heavily doped *n*-type layers. However, uncertainties in the surface concentration and impurity profile of diffused layers prohibit a quantitative analysis of the results. The purpose of the present work is to reinvestigate the piezoresistive properties of heavily doped *n*-type silicon using uniformly doped samples so that the results can be quantitatively analyzed to give information on the band structure and mobility anisotropy.

The piezoresistance effect has been measured on oriented silicon samples having impurity concentrations from approximately 1×10^{15} to 1×10^{20} cm^{-3}. The coefficient π_{11} was measured between 77 and 370°K and the coefficient π_{44} and the hydrostatic pressure coefficient were measured at 300°K. The piezoresistance effect has previously been investigated at the lower impurity concentrations,[22,23] but it is useful in the analysis to include these concentrations in the present work in order to determine the complete behavior of the piezoresistance effect in one set of experiments. The results are analyzed in terms of the electron-transfer mechanism and possible explanations for the discrepancies between the measured results and the predictions of the electron-transfer mechanism are given.

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[1] G. L. Pearson and J. Bardeen, Phys. Rev. **75**, 865 (1949).
[2] F. J. Morin and J. P. Maita, Phys. Rev. **96**, 28 (1954).
[3] G. Backenstoss, Phys. Rev. **108**, 1416 (1957).
[4] K. B. Wolfstern, Phys. Chem. Solids **16**, 1 (1960).
[5] R. A. Logan, J. F. Gilbert, and F. A. Trumbore, J. Appl. Phys. **32**, 131 (1961).
[6] P. W. Chapman, O. N. Tufte, J. David Zook, and Donald Long, J. Appl. Phys. **34**, 3291 (1963).
[7] For a review see S. H. Koenig, in *Proceedings of the International Conference on Semiconductor Physics, Exeter, 1962* (The Institute of Physics and The Physical Society, London, 1962), p. 5.
[8] W. G. Spitzer and H. Y. Fan, Phys. Rev. **106**, 882 (1957).
[9] M. Cardona, W. Paul, and H. Brooks, Helv. Phys. Acta **33**, 329 (1960).
[10] L. E. Howarth and J. F. Gilbert, J. Appl. Phys. **34**, 236 (1963).
[11] D. Geist, Naturwiss. **45**, 33 (1958).
[12] E. Sonder and D. K. Stevens, Phys. Rev. **110**, 1027 (1958).
[13] P. Keesom and G. Seidel, Phys. Rev. **113**, 33 (1959).
[14] R. H. Parmenter, Phys. Rev. **104**, 22 (1956); M. Lax and J. C. Phillips, *ibid.* **110**, 41 (1958).
[15] V. L. Bonch-Bruevich, Fiz. Tverd. Tela **4**, 2260 (1962) [English transl.: Soviet Phys.—Solid State **4**, 1953 (1963)].
[16] E. M. Conwell and D. W. Levinger, *Proceedings of the International Conference on Semiconductor Physics, Exeter, 1962* (The Institute of Physics and The Physical Society, London, 1962), p. 227.
[17] P. A. Wolff, Phys. Rev. **126**, 405 (1962).
[18] E. O. Kane, in *Proceedings of the International Conference on Semiconductor Physics, Exeter, 1962* (The Institute of Physics and the Physical Society, London, 1962), p. 252; Phys. Rev. **131**, 79 (1963).
[19] R. A. Logan and A. G. Chynoweth, Phys. Rev. **131**, 89 (1963).
[20] M. Pollack, Phys. Rev. **111**, 798 (1958).
[21] O. N. Tufte and E. L. Stelzer, J. Appl. Phys. **34**, 313 (1963).
[22] C. S. Smith, Phys. Rev. **94**, 42 (1954).
[23] F. J. Morin, T. H. Geballe, and C. Herring, Phys. Rev. **105**, 525 (1957).

II. DISCUSSION OF THE PIEZORESISTANCE EFFECT IN DEGENERATE SILICON

The theory of the piezoresistance effect in a semiconductor having a many-valley type band structure has been explained by Herring[24] in terms of the electron-transfer mechanism. For the energy-band structure of n-type silicon, which consists of six energy minima located on the [100] reciprocal axes, the theoretical expressions for the low-stress piezoresistance coefficients are[25,26]

$$\pi_{11}=\pi_{11}(0)\frac{\mathfrak{F}_{j-1/2}(\eta)}{\mathfrak{F}_{j+1/2}(\eta)}, \tag{1}$$

where

$$\pi_{11}(0)=\tfrac{2}{3}\frac{\Xi_\mu}{(C_{11}-C_{12})kT}\frac{K-1}{2K+1}, \tag{2}$$

$$\pi_{12}=-\pi_{11}/2, \tag{3}$$

and

$$\pi_{44}=0. \tag{4}$$

In these equations, Ξ_μ is the deformation potential, C_{11} and C_{12} elastic constants, $K\equiv\mu_\perp/\mu_\parallel$ the mobility anisotropy within a valley, η the reduced Fermi level defined as the Fermi energy E_f divided by kT, and $\mathfrak{F}_k(\eta)$ are the Fermi-Dirac integrals defined by[27,28]

$$\mathfrak{F}_k(\eta)=\frac{1}{\Gamma(k+1)}\int_0^\infty\frac{\epsilon^k d\epsilon}{1+e^{\epsilon-\eta}}, \tag{5}$$

where ϵ is the energy above the band edge in units of kT. It is assumed in these expressions that the energy dependence of the relaxation time τ can be written in the form $\tau=\tau_0E^j$.

In lightly doped silicon where classical statistics are applicable, the electron-transfer effect predicts that $\pi_{11}=\pi_{11}(0)$ and is independent of impurity concentration and varies with temperature as the reciprocal of the absolute temperature, providing Ξ_μ and K are independent of impurity concentration and temperature. However, in this case, intervalley scattering can significantly alter the predictions of the electron-transfer mechanism at temperatures above approximately 150°K.[23,24]

In silicon doped to impurity concentrations where statistical degeneracy occurs, the piezoresistance effect is a useful tool for the study of energy-band structure and carrier scattering effects since it is a function of η, K, and j. However, since it is a function of several parameters, the interpretation of the piezoresistance results depends on the separation of the effects due to each parameter. In this section, the dependence of the piezoresistance effect on η, K, and j, and the degree to which effects due to each may be isolated will be discussed.

From Eq. (1) it is seen that in the region of statistical degeneracy, π_{11} depends on η through the ratio of the Fermi-Dirac integrals. Since these integrals cannot be expressed as analytic functions for arbitrary values of η, it is useful to examine the case of large degeneracy ($\eta\gtrsim5$) where the expansion[28]

$$\mathfrak{F}_k(\eta)=\frac{\eta^{k+1}}{\Gamma(k+2)} \tag{6}$$

may be used. In this case, Eq. (1) may be rewritten as

$$\pi_{11}=\pi_{11}(0)(j+\tfrac{3}{2})/\eta. \tag{7}$$

If the conduction band is assumed to be parabolic, the relation between η and the electron concentration n in the highly degenerate limit is given by

$$\eta=Bn^{2/3}, \tag{8}$$

where

$$B=(3/\pi)^{2/3}(h^2/8m_0^*kT) \tag{9}$$

and m_0^* is the density-of-states effective mass.

Substitution of Eq. (8) into Eq. (7) gives

$$\pi_{11}=\pi_{11}(0)(j+\tfrac{3}{2})/Bn^{2/3}. \tag{10}$$

If ln π_{11} is plotted against ln n at a constant temperature, Eq. (10) predicts a linear dependence with a slope of $-\frac{2}{3}$ if the band is parabolic. It should be noted that the slope is independent of the values of j, $\pi_{11}(0)$, and m_0^*, providing they are constant or slowly vary with n in the region of large statistical degeneracy.

In the Appendix it is shown that if the band is nonparabolic such that the effective mass increases with energy above the conduction-band edge, the slope of a logarithmic plot of π_{11} against n will be smaller than $-\frac{2}{3}$ with the exact value depending on the degree of nonparabolicity and the value of j. Similarly, if the effective mass decreases with increasing energy, the slope will be larger than $-\frac{2}{3}$. It is also shown in the Appendix that the slope is quite sensitive to nonparabolic effects with a change in m^* of 10% in the most heavily doped sample being detectable. It can therefore be concluded that if the slope is $-\frac{2}{3}$, the band is parabolic or at least the total number of states below the Fermi level is the same as that in a parabolic band. If the slope is not equal to $-\frac{2}{3}$, the degree of nonparabolicity cannot be determined from the piezoresistance measurements directly since the slope also depends on the value of j in this case.

In the region of statistical degeneracy, π_{11} also depends on j through the Fermi-Dirac integrals. In the previous paragraph it was shown that in the region of large η, the value of j will directly affect the absolute value of π_{11} in the region of large degeneracy. Thus, if the experimentally measured results have a slope of $-\frac{2}{3}$, the value of j could be obtained by fitting the magnitude of the calculated and measured π_{11} values in the highly degenerate region providing m_0^* is constant, or its

[24] C. Herring, Bell System Tech. J. **34**, 237 (1955).
[25] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).
[26] See also R. W. Keyes, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1960), Vol. 11.
[27] R. B. Dingle, Appl. Sci. Res. Sect. B **6**, 225 (1956).
[28] See also J. S. Blakemore, *Semiconductor Statistics* (Pergamon Press, Inc., New York, 1962).

dependence on concentration known, and $\pi_{11}(0)$ is independent of impurity concentration. Since there is some disagreement in the literature as to the dependence of m^* on impurity concentration,[8–13] it will be assumed to be constant in the present discussion. However, in n-type silicon $\pi_{11}(0)$ will not be independent of impurity concentration due to its sensitivity to K which is a function of impurity concentration.

The sensitivity of $\pi_{11}(0)$ to the value of K may be obtained by differentiation of Eq. (1) and is given by[29]

$$\frac{\delta\pi(0)}{\pi(0)}=\frac{3}{2K-1-(1/K)}\frac{\Delta K}{K}. \tag{11}$$

Since $K\approx5$ in lightly doped silicon, $\Delta\pi(0)/\pi(0)=\frac{1}{3}\Delta K/K$, so that $\pi_{11}(0)$ will be sensitive to K changes greater than three times the experimental error in $\pi_{11}(0)$. With increased doping, the value of K decreases, making $\pi_{11}(0)$ even more sensitive to the value of K. Thus, the value of j cannot be uniquely determined from the piezoresistance results, since the dependence of K on impurity concentration and temperature is not known in heavily doped silicon. However, the experimental results can be analyzed to the point where certain combinations of j and K values can be shown to be consistent with the experimental results.

Throughout this paper, the assumption that N, the impurity concentration, is equal to n, the electron concentration, has been made when calculating η values. If the impurity activation energy is zero or too small to cause an appreciable deionization at temperatures down to 77°K, this assumption is correct. In n-type silicon, deionization effects begin to occur at 77°K if the impurity concentration is less than 2×10^{18} cm^{-3}. However, the effect of statistical degeneracy on the π_{11} values is small below this concentration, so any error involved in assuming $N=n$ will have a negligible effect on the calculated π_{11} values.

III. EXPERIMENTAL PROCEDURE

A. Sample Preparation

Bridge-type samples of the form shown in Fig. 1 were ultrasonically cut from single crystals grown by the Czochralski method. Some of the crystals were grown in our laboratory, while others were purchased from commercial suppliers. The crystals were doped with either phosphorus or arsenic with a majority being phosphorus doped. The samples were cut with the sample axis along either a [100], [110], or [111] crystallographic direction. The samples were lapped and etched after cutting. The areas to which electrical contacts were to be made were masked during the etching and subsequently electroless[30] nickel plated on the lapped surface. Electrical leads were soldered to the nickel-plated areas.

[29] An expression similar to this but simplified for the case where $K\gg1$ is given in Ref. 20.

[30] M. V. Sullivan and J. H. Eigler, J. Electrochem. Soc. **104**, 226 (1957).

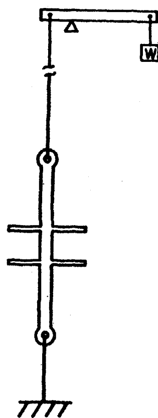

FIG. 1. Schematic drawing of sample geometry and the method of applying tensile stress to the sample.

Slices cut from the most heavily doped crystals were etched in a Dash[31] etch to check for large concentrations of dislocations or other crystalline imperfections. In one crystal, precipitates that were readily removed by the Dash etch were observed.[32] Samples cut from this crystal gave smaller π_{11} values and larger π_{44} values than samples cut from crystals free from these imperfections. Microscopic examination of the voids left by the removal of the precipitates indicates that the precipitated regions are roughly rectangular in shape with the long dimension of the rectangle along a [100] crystallographic axis. The current flow pattern in the samples is probably distorted by the presence of these defects, which accounts for their influence on the piezoresistive properties. Two other crystals were found to be free of these defects and the piezoresistance results obtained on samples cut from these two crystals are identical to within experimental error, indicating the true piezoresistance effect is being observed. As an additional check, at least one heavily doped sample of each orientation was Dash etched after completion of the experimental measurements to verify the absence of any defects.

B. Measurement of the Impurity Concentration

The impurity concentration was evaluated from the sample resistivity using the curves of Irvin[33] relating resistivity and impurity concentration. Hall-coefficient measurements were also made on most of the samples and the impurity concentration was evaluated from the relation $n=r/Re$, where R is the Hall coefficient and $r\equiv\mu_H/\mu$, the Hall-coefficient factor. If r is set equal to unity, the impurity concentration values determined from the Hall-coefficient measurements are approximately 20% less than the values obtained from the resistivity measurements over most of the impurity concentration range.

[31] W. C. Dash, J. Appl. Phys. **27**, 1193 (1956).

[32] Recently, similar effects have also been observed in silicon heavily doped with gallium. T. Iizuka, M. Kikuchi, and K. Kanasaki, Japan J. Appl. Phys. **2**, 196 (1963).

[33] J. C. Irvin, Bell System. Tech. J. **41**, 387 (1962).

The uniformity of the impurity concentration in the samples was checked by comparing the Hall coefficient values at the two sets of Hall probes. A variation of the order of 5% or less was present in the samples used in this investigation.

C. Measurement Apparatus and Procedure

A tensile stress is applied to the sample by means of cords passing through holes in the enlarged sample ends as shown in Fig. 1. On the lower end, the cord is fastened to the sample holder, while on the upper end the cord extends about 2 cm and is then fastened to a thin brass rod. The brass rod extends out of the sample holder and is fastened to a beam balance. Stress is applied to the sample by applying weights to the balance. Stresses in the range from 10^7 to 10^8 dyn/cm^2 were used in this investigation. Other methods of holding the sample, such as having collars fit around the enlarged sample ends and having pins go through the holes in the ends, were also tried and found to be comparable with the method used.

The sample holder consisted of a large copper block surrounded by a copper can. The sample holder was suspended in or placed over a suitable temperature bath and temperature control was maintained by means of a heater wound around the copper can. For temperatures below room temperature, liquid nitrogen was used as a temperature bath while for temperatures above room temperature, a glycerine bath was used. Below room temperature, the sample was in a helium-gas atmosphere to prevent the formation of frost. All temperatures were measured with a copper-constantan thermocouple.

The isothermal piezoresistance coefficients were measured by a conventional dc method utilizing a Rubicon B potentiometer, a dc amplifier and recorder. The unstrained sample voltage is balanced out on the potentiometer and the galvanometer output of the potentiometer connected to the amplifier and recorder. The change in voltage due to the applied stress is displayed on the recorder to check the stability of the signal. The potentiometer is then rebalanced using the amplifier-recorder as a null device to eliminate possible errors due to errors in the calibration of the amplifier-recorder system. A linear relation between the voltage change and the applied stress was found on all samples. The apparatus used to measure the hydrostatic pressure coefficient is similar to that described by Long.[34] The isothermal hydrostatic pressure coefficient was found to be independent of pressure for pressures in the range 10^8 to 10^9 dyn/cm^2. The accuracy of the π_{11} measurements is $\pm 3\%$, while the limits on the hydrostatic pressure coefficients and π_{44} are $\pm 10\%$.

The relations between the coefficients measured on [100], [110], and [111] oriented samples and the phenomenological coefficients π_{11}, π_{12}, and π_{44} are given in the literature[22,35] and will not be repeated here. In the present work, the coefficient π_{11} was measured directly and the coefficient π_{12} was measured at room temperature by combining π_{11} with the hydrostatic pressure coefficient. The π_{44} coefficient was evaluated at room temperature by combining measurements of the longitudinal piezoresistance on [111] oriented samples with the hydrostatic-pressure coefficient measured on the same sample. Checks were also made from measurements on [100] and [110] samples combined with the hydrostatic-pressure coefficient with satisfactory agreement being found in all cases. Since the hydrostatic-pressure coefficient could only be measured in the vicinity of room temperature with our present apparatus, π_{44} could only be evaluated at room temperature.

A correction for the change in dimensions of the sample due to the applied stress has been made in all of the experimental results.

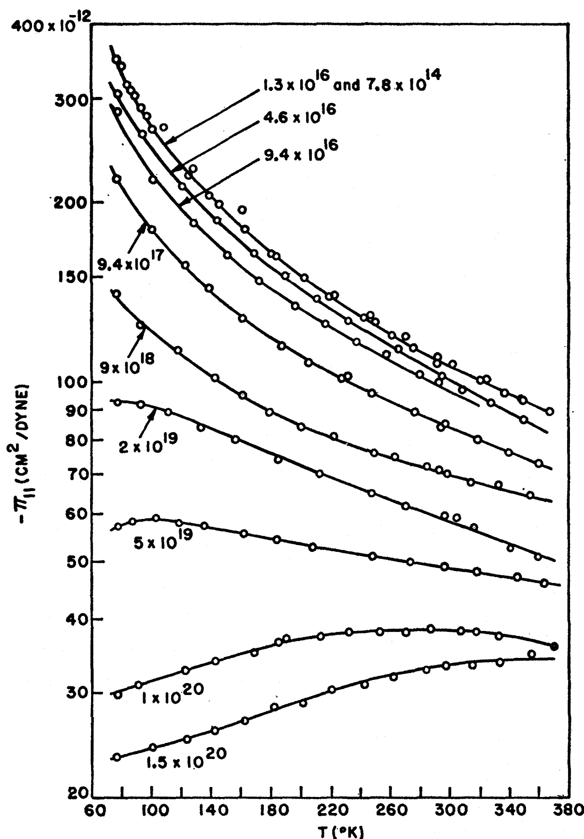

FIG. 2. The measured variation of π_{11} with temperature in n-type silicon. The solid lines are drawn to give a best fit to the experimental data.

IV. EXPERIMENTAL RESULTS

The variation of π_{11} with impurity concentration and temperature is shown in Figs. 2 and 3. The direct results

[34] D. Long, Phys. Rev. **99**, 388 (1955).

[35] W. P. Mason and R. N. Thurston, J. Acoust. Soc. Am. **29**, 1096 (1957).

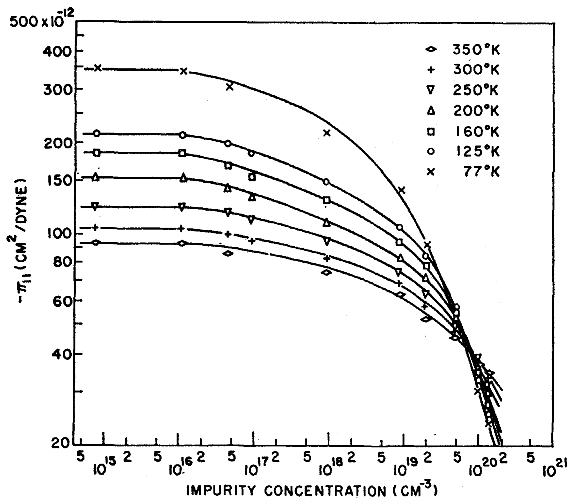

FIG. 3. The dependence of π_{11} on impurity concentration with the temperature as a parameter. The points on this curve are obtained from the solid lines on Fig. 2.

of the measurements are shown in Fig. 2. In Fig. 3, these results have been replotted to show the variation of π_{11} with impurity concentration at fixed temperatures which is the form used in the discussion of the results in the next section. These results show two unusual features which are the basis for much of the discussion. First, a noticeable decrease in π_{11} occurs at impurity concentrations as low as 4×10^{16} cm^{-3}, whereas the statistical degeneracy effect based on an undistorted parabolic band model predicts that π_{11} should be independent of impurity concentration up to concentrations of approximately 1×10^{18} cm^{-3}. Secondly, in the samples having the highest impurity concentrations, the value of π_{11} goes through a maximum as a function of temperature and decreases noticeably with decreasing temperature. This also is in contradiction to the predictions of the statistical degeneracy effect in Eq. (1) which predicts that π_{11} should be approximately independent of temperature in the most heavily doped samples. All of the results shown in Figs. 2 and 3 were obtained with phosphorus-doped samples. One arsenic-doped sample having an impurity concentration of 5×10^{19} cm^{-3} was also investigated and the π_{11} results fit in both magnitude and temperature dependence with the results in Fig. 2.

The results of isothermal hydrostatic-pressure coefficient measurements at 300°K are given in Fig. 4. The hydrostatic pressure coefficient is seen to remain small at all concentrations, but it does go through a minimum at an impurity concentration of approximately 1×10^{17} cm^{-3}. For a sample having an impurity concentration of approximately 4×10^{14} cm^{-3}, Smith[22] reported a value of 5.7×10^{-12} which is approximately twice as large as the present results. The reason for this discrepancy is not understood. At both the highest and the lowest concentrations, a definite difference between phosphorus- and arsenic-doped samples is seen. At high impurity concentrations, a difference in the magnitude[5,6] and temperature dependence[6] of the resistivity between arsenic- and phosphorus-doped silicon has previously been observed, indicating the scattering is somewhat dependent on the type impurity atom in this concentration range. In view of this, the dependence of the pressure coefficient on the type impurity atom is not unexpected, but a quantitative understanding of the origin of the effect will undoubtedly have to await a quantitative understanding of the scattering mechanisms in heavily doped silicon.

The relatively small value of the hydrostatic-pressure coefficient shows that $\pi_{12}\approx-\pi_{11}/2$ at 300°K over the entire impurity concentration range. This is in agreement with the predictions of the electron-transfer mechanism for n-type silicon [Eq. (3)].

The measured values of π_{44} at 300°K are shown in Fig. 5. The value of π_{44} in the most pure sample compares favorably with the value of -13.6×10^{-12} cm²/dyn reported by Smith[22] in lightly doped silicon. A definite increase in π_{44} at impurity concentrations greater than 5×10^{19} cm^{-3} is seen in the experimental results. It should also be noted that a 5×10^{19} cm^{-3} arsenic-doped sample gave very nearly the same π_{44} value as a phosphorus-doped sample, indicating the π_{44} values are not strongly dependent on the type of impurity atom used. Arsenic-doped samples having impurity concentrations of 1×10^{20} cm^{-3} or greater are not available, so the increase in π_{44} at the highest concentrations could not be investigated in arsenic-doped samples.

The magnitude and temperature dependence of π_{11}, π_{44}, and the hydrostatic-pressure coefficient are all in qualitative agreement with the previous results[21] measured on diffused layers.

V. ANALYSIS OF RESULTS AND DISCUSSION

A. Dependence of π_{11} on Impurity Concentration

In the analysis of the π_{11} coefficient, it is convenient to divide the discussion into two parts by first considering the dependence of π_{11} on impurity concentration at

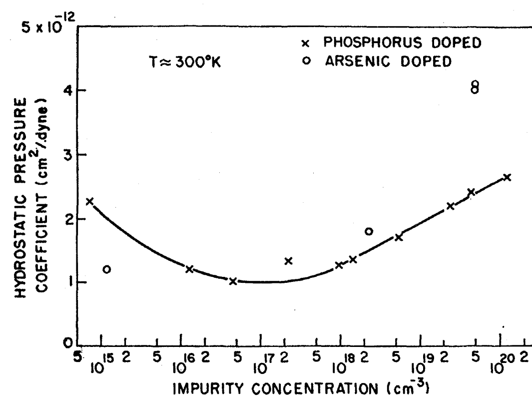

FIG. 4. The measured variation of the hydrostatic-pressure coefficient with impurity concentration in n-type silicon at 300°K.

fixed temperatures, which is done in this section, and then considering the dependence of π_{11} on temperature in the following section.

In order to investigate the parabolic nature of the conduction band in silicon, the experimental variation of π_{11} with impurity concentration at 77°K, shown in Fig. 3, has been replotted in Fig. 6 along with the predictions of Eq. (1). For the calculated curves, the η values obtained by assuming a parabolic band with a density-of-states effective mass of[36] 1.06 m_0 were used. The $\pi_{11}(0)$ values were set equal to the π_{11} values measured at 77°K on the purest samples. The 77°K results are used since theory and experiment are to be compared in the region of large η and also because any intervalley scattering effects[24,25] which will change the predictions of Eq. (1) are not present at this temperature. The calculated curves in Fig. 6 show graphically the results predicted by Eq. (10) in that in the region of large degeneracy, the slopes of the calculated curves are equal to $-\frac{2}{3}$ and are independent of the value of j. By varying j, the calculated π_{11} values may be adjusted to fit the magnitude of the experimental values. In the region of large degeneracy ($\eta>5$), the experimental π_{11} values in Fig. 6 are seen to lie on a line having a slope of $-\frac{2}{3}$. While it is possible that the dependence of $\pi_{11}(0)$, m^*, or j on impurity concentration may just offset any change in the $-\frac{2}{3}$ slope due to the band being nonparabolic, a very fortuitous balance would be required for the experimental results to follow so closely a $-\frac{2}{3}$ slope. It is therefore concluded that the values of η calculated by assuming a parabolic band are correct in heavily doped n-type silicon.

Since the piezoresistance effect measures only the degree of degeneracy, tailing effects and changes in curvature of the band below the Fermi level will not be observable if the total number of states below the Fermi level is very nearly the same as for a parabolic band. It is therefore concluded that the density of states in the band tail for a given doping level must be small compared to the total density of states below the Fermi level or that the states in the tail come from near the bottom of the undeformed conduction band with a subsequent decrease in states there so that the total number of states below the Fermi level is the same as for a parabolic band.

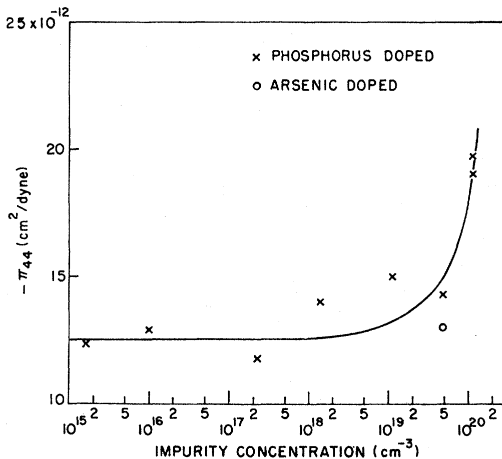

FIG. 5. The measured variation of the coefficient π_{44} with impurity concentration in n-type silicon at 300°K.

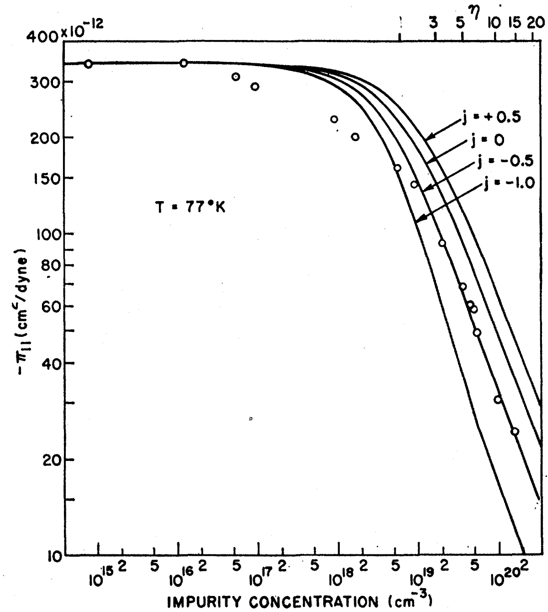

FIG. 6. The dependence of π_{11} on impurity concentration calculated from Eq. (1) by assuming the conduction band is parabolic with a density-of-states effective mass of 1.06 m_0. The points are the experimental data from Fig. 3 with several additional points added at high impurity concentrations.

The present results are in agreement with theoretical predictions[17] of the effect of heavy doping on the energy-band structure of semiconductors which indicate that the curvature of the band away from the edge is unchanged by doping. The present results are also in agreement with recent free carrier-absorption experiments[10] in which the effective mass was found to be unchanged by heavy doping, indicating the conduction band is parabolic near the Fermi level.

Since the relation between η and n appears to be correctly given by assuming a parabolic conduction band, the dependence of the magnitude of π_{11} on impurity concentration predicted by Eq. (1) can be combined with the experimental π_{11} results to provide information on the value of j and the dependence of $\pi_{11}(0)$ on impurity concentration. In the present analysis, specific values for j will be assumed and the dependence of π_{11} on impurity concentration at constant temperatures calculated from Eq. (1) with $\pi_{11}(0)$ set equal to its value in the purest samples. Any discrepancy between the calculated curve and the experimental results is then attributed to a change in $\pi_{11}(0)$ with impurity concentration. A small variation in $\pi_{11}(0)$

[36] C. J. Rauch, J. J. Stickler, H. J. Zeiger, and G. S. Heller, Phys. Rev. Letters 4, 64 (1960).

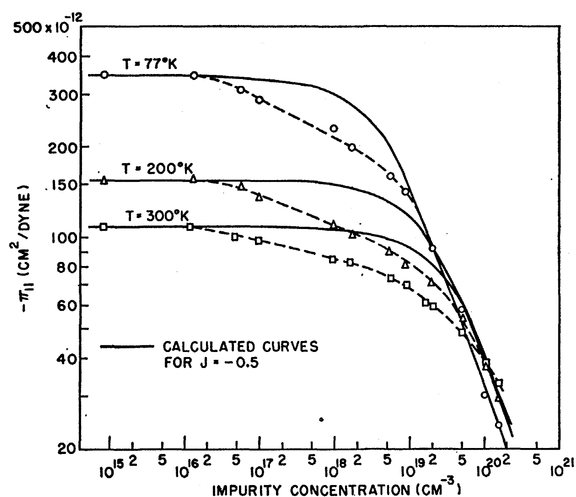

FIG. 7. A comparison of the π_{11} values calculated for $j=-0.5$ with the experimental results. The dashed lines are best fits to the experimental data points.

with impurity concentration will result from the dependence of the elastic constants on electron concentration. However, the largest change in the elastic constant $(C_{11}-C_{12})$ is[37] only approximately 8% so this effect has been neglected and the entire change in $\pi_{11}(0)$ attributed to the change in K as discussed in Sec. II. This procedure is justified in the present case since the results obtained are only of a qualitative nature.

The two j values that have been considered are $j=-0.5$, which was seen to give good agreement between the measured and calculated π_{11} values at 77°K in the region of heavy doping in Fig. 6, and $j=0$, which was the assumption used by Pollak[20] in his analysis of the piezoresistance results in n-type germanium. The π_{11} values calculated from Eq. (1) with $j=-0.5$ and $\pi_{11}(0)$ equal to the measured π_{11} values on the purest samples are compared with the experimental results in Fig. 7. For clarity, curves are shown for only three temperatures in Fig. 7 although similar results were obtained at all temperatures investigated. At 77°K, the calculated and experimental values agree very well for impurity concentrations greater than 1×10^{19} cm^{-3} indicating that $\pi_{11}(0)$ is the same at both the low and the high impurity concentrations if $j=-0.5$. At higher temperatures, the divergence between the calculated and experimental values becomes greater until at 300°K agreement is found only at the highest impurity concentrations.

The dependence of K on impurity concentration and temperature can be evaluated by assuming $K=5$ at all temperatures in the purest samples[38,39] and, from the experimental values of $\pi_{11}[=\pi_{11}(0)]$ in the purest samples, evaluating the quantity $2\Xi_u/[3(C_{11}-C_{12})kT]$ at each temperature [see Eq. (2)]. The value of K required to give the experimentally measured $\pi_{11}(0)$ value at each concentration can then be evaluated. The K values obtained by this procedure for $j=-0.5$ are shown in Fig. 8. In the very lightly doped region, the K values are only approximate, since with an accuracy of $\pm3\%$ in the π_{11} values, K must decrease to 4.5 before a change will be observable in π_{11}. For impurity concentrations below 1×10^{17} cm^{-3}, the results show the expected behavior in that the mobility anisotropy decreases with increasing impurity concentration and decreasing temperature due to increased ionized impurity scattering.[40,41] In the medium doping range, this trend reverses and the mobility anisotropy again increases with the lower temperatures showing the larger anisotropy, until at the highest concentrations the mobility anisotropy is approximately the same as in pure silicon.

The dependence of K on impurity concentration and temperature obtained by the same analysis except with $j=0$ is also shown in Fig. 8. In the nondegenerate region of doping, the K values are independent of j as expected from Eqs. (1) and (2). In the region of heavy doping, the assumption of $j=0$ leads to the result that K is independent of temperature and impurity concentration.

From the analysis of the piezoresistance results it is seen that in the heavily doped region, the behavior of K

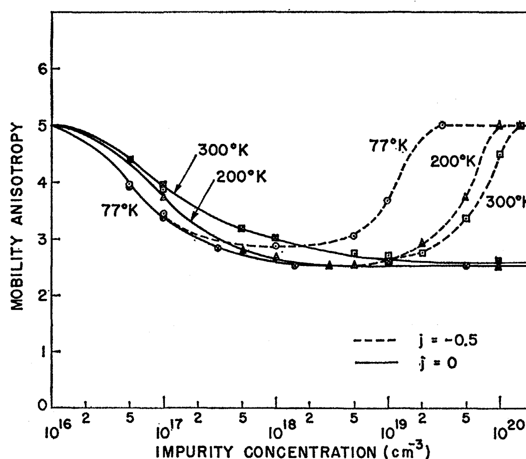

FIG. 8. The variation of the mobility anisotropy with temperature and impurity concentration. The dashed curve is the result of assuming $j=-0.5$ in the calculated results. For the solid lines, the assumption $j=0$ was used.

[37] N. G. Einspruch and P. Csavinszky, Appl. Phys. Letters **2**, 1 (1963).

[38] The effective mass ratio $m_{\parallel}/m_{\perp}$ is approximately 5 in n-type silicon [R. N. Dexter, H. J. Zeiger, and B. Lax, Phys. Rev. **104**, 637 (1956)]. Since the relaxation time anisotropy is near unity in pure silicon over the temperature range of interest here [D. Long, Phys. Rev. **120**, 2024 (1960)], the mobility anisotropy is assumed to be 5.

[39] For a review of K values determined from magnetoresistance measurements, see M. Glicksman, in *Progress in Semiconductors*, edited by A. F. Gibson, P. Aigrain, and R. E. Burgess (John Wiley and Sons Inc., New York, 1958), Vol. 3, p. 20.

[40] L. J. Neuringer and D. Long, Bull. Am. Phys. Soc. **8**, 218 (1963).

[41] F. Ham, Phys. Rev. **100**, 1251 (1955).

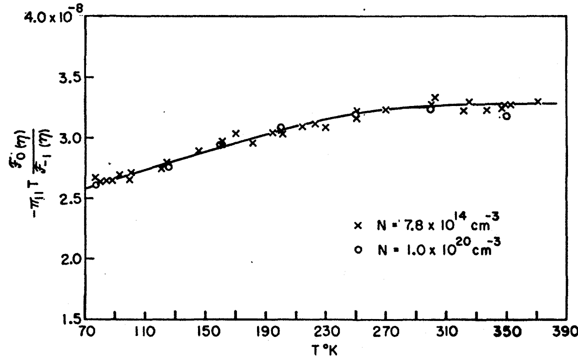

FIG. 9. The variation of the normalized π_{11} coefficients with temperature for two samples having a wide difference in impurity concentration. The assumption that $j=-0.5$ was made in normalizing the π_{11} values in the sample doped to 1×10^{20} cm^{-3}.

is strongly dependent on the value of j. Since the value of j cannot be determined from the low-stress piezoresistance results, one can only determine for assumed values of j the behavior of K that is consistent with the piezoresistance results. It has also been assumed in the analysis that a single value of j is applicable over the entire range of temperature and impurity concentration whereas in reality, the value of j will probably not be constant.

At this point, it is useful to examine the results of other experiments on heavily doped silicon which have a bearing on the values of j or K. A value of j can be obtained from the temperature dependence of the electrical conductivity. Analysis of previous electrical conductivity results[6] shows that in the region of heavy doping ($10^{19}-10^{20}$ cm^{-3}), a negative j value is required to fit the experimental results in the temperature range of interest here, with the value ranging from -0.5 to -1.0, depending on the concentration. In the medium doping range, the value of j varies from negative to positive and back to negative with increasing temperature, similar to the results reported by Pollak[20] for n-type germanium. However, the j values determined from the temperature dependence of the resistivity in heavily doped semiconductors are probably at least partly due to a change in the screening radius of the ionized impurities with temperature[6,7,42] rather than a change in relaxation time with carrier energy. Therefore, a quantitative application of these values should not be made. However, at high impurity concentrations, the temperature dependence of the conductivity indicates a negative value for j. Magnetoresistance measurements[43] have also been used to determine K in samples having impurity concentrations in the range 1×10^{15} to 5×10^{19} cm^{-3}. The K values deduced from these measurements follow qualitatively the $j=-0.5$ piezoresistance results in that at the higher impurity concentrations, K increases to near 5 and the low temperatures give larger K values than the high temperature. On the basis of these results it is concluded that j is probably negative in heavily doped n-type silicon and the K values corresponding to $j=-0.5$ are qualitatively correct. A more quantitative interpretation of these results will be possible only if the nature of the scattering mechanisms contributing to the empirical parameter j is understood.

For a tensor relaxation time,[25] K may be written as

$$K\equiv\frac{\mu_\perp}{\mu_{\parallel}}=\frac{m_{\parallel}}{m_\perp}\Big/\frac{\tau_{\parallel}}{\tau_\perp}, \qquad (12)$$

and the dependence of K on impurity concentration in Fig. 8 compared with the behavior of $\tau_{\parallel}/\tau_\perp$ predicted by Samoilovich, Korenblit, and Dakhovskii (SKD).[44] Since the total number of states below the Fermi level has been shown to be correctly given by assuming a parabolic conduction band, the ratio of $m_{\parallel}/m_\perp$ is assumed to be independent of impurity concentration. The decrease in $\tau_{\parallel}/\tau_\perp$ from its value of near four[40,41,44] in lightly doped silicon to a value near unity in heavily doped silicon, as well as the temperature dependence of $\tau_{\parallel}/\tau_\perp$ for $j=-0.5$, are qualitatively in agreement with the behavior extrapolated from the work of SKD.[44] A quantitative comparison cannot be made, however, since the calculations of SKD are not applicable in very heavily doped silicon.

B. Dependence of π_{11} on Temperature

The second unexpected feature of the experimental results shown in Fig. 2 is the temperature dependence of π_{11} in the most heavily doped samples at the lower temperatures. A clue to this behavior is given by the fact that in both our measurements and in the previously reported results of Morin, Geballe, and Herring,[23] the temperature dependence of π_{11} at the lower temperatures is not as strong as the $1/T$ dependence predicted by the electron-transfer mechanism. The present results indicate that the mechanism that causes the deviation from a $1/T$ dependence in lightly doped silicon is also present in heavily doped silicon and causes the decrease in π_{11} with decreasing temperature. This can be quantitatively seen in Fig. 9 where the normalized coefficient $\pi_{11}\ T[\mathfrak{F}_{j+1/2}(\eta)]/[\mathfrak{F}_{j-1/2}(\eta)]$, with $j=-0.5$, is plotted against temperature for a very lightly doped and a very heavily doped sample. For the lightly doped sample, the ratio of the Fermi-Dirac functions is of course unity, so the points shown in Fig. 9 are the experimental data points. For the heavily doped samples, the product $T\mathfrak{F}_0(\eta)/\mathfrak{F}_{-1}(\eta)$ is approximately constant over this temperature range, so the points for this sample in Fig. 9 are the experimental π_{11} values multiplied by a temperature-independent constant. Thus it is seen that both samples have the

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[42] W. Bernard, H. Roth, and W. D. Straub, Phys. Rev. **132**, 33 (1963).

[43] Unpublished data of the authors.

[44] A. G. Samoilovich, I. Ya. Korenblit, and I. V. Dakhovskii, Dokl. Akad. Nauk SSSR **139**, 355 (1961) [English transl.: Soviet Phys.—Doklady **6**, 606 (1962)].

same temperature dependence of the normalized π_{11} coefficients, indicating the same mechanism is responsible for this behavior in both samples. The excellent agreement in magnitude of the normalized π_{11} coefficients in Fig. 9 is a result of using $j=-0.5$ and is not considered significant since it depends on the exact value of j which is not known. Also, the elastic constant $C_{11}-C_{12}$ changes by[37] 5–8% due to doping, and this should cause the magnitudes to be different on the two samples. However, the temperature dependences of the normalized coefficients will remain similar for all reasonable j values.

From Eqs. (1) and (2), the normalized π_{11} coefficient is given by

$$\pi_{11}T\frac{\mathfrak{F}_0(\eta)}{\mathfrak{F}_{-1}(\eta)}=\frac{2}{3}\frac{\Xi_\mu}{k(C_{11}-C_{12})}\left(\frac{K-1}{2K+1}\right). \qquad (13)$$

The two impurity concentrations used in Fig. 9 were chosen because the factor $[(K-1)/(2K+1)]$ in Eq. (13), which will be referred to as the K factor, is very nearly independent of temperature for these concentrations. In lightly doped silicon, magnetoresistance measurements[39] indicate a K change over this temperature interval of approximately 10%, giving a change in the K factor of only 3%. In the heavily doped sample, the previous discussion indicates that K is independent of temperature at the highest impurity concentrations for both $j=-0.5$ and $j=0$. Physically, it would be expected that K will be independent of temperature in the case of large degeneracy since the scattering mechanism for ionized impurity scattering will depend on the velocity of the carriers and the velocity, in this case, is determined largely by the Fermi energy rather than the thermal energy. Also, it would be very fortuitous if the temperature dependence of the K factor is the same for both samples since the scattering mechanism is very different. It is therefore concluded that the K factor is not the source of the temperature dependence in Fig. 9. Another factor that has not yet been considered is the effect of intervalley scattering[23,24] on the piezoresistance effect. However, this will not explain the anomalous temperature dependence of the π_{11} coefficient in even the lightly doped sample, since $\pi_{11}T$ should be independent of temperature at the lowest temperature and increase at the higher temperatures. Also, the contribution of intervalley scattering would not be expected to be independent of impurity concentration over such a wide range of concentration. The temperature dependence of the elastic constants over this temperature range is only 1.5% in lightly doped[45] and 3% in heavily doped[37] silicon. It is therefore concluded that the temperature dependence of the normalized π_{11} coefficients in Fig. 9 is due to the variation of the deformation potential constant with temperature. Of all the factors considered, this is the only one that can give a change of the required magnitude and yet be independent of impurity concentration over five orders of magnitude in concentration.

If the deformation potential is made a function of temperature of the form

$$\Xi_\mu=\Xi_{\mu0}(1+\alpha T), \qquad (14)$$

the coefficient α is found to be 1.4×10^{-3} °K^{-1} at the lower temperatures. For comparison, Fritzsche[46] has reported an α value in n-type germanium of -0.6×10^{-3} °K^{-1}. The deformation potential values measured by Schmidt-Tiedemann[47] in n-type silicon do not show the temperature dependence predicted by the present results, but the experimental error in his values is large enough to mask a temperature dependence of the magnitude found in the present work.

From Fig. 9, it appears that α goes to zero at the higher temperatures. In the lightly doped sample it is not possible to draw any quantitative conclusions on the temperature dependence of π_{11} at higher temperatures, since the effects of intervalley scattering on the piezoresistance have not been taken into account. In the absence of any effect directly attributable to intervalley scattering, no further analysis is felt to be warranted. The reason for the decrease in the temperature dependence at the higher temperatures in the heavily doped sample is not understood.

C. π_{44} Coefficient

The rather large magnitude of the π_{44} coefficient at impurity concentrations below 5×10^{19} cm^{-3} is hard to understand since the electron-transfer mechanism predicts this coefficient to be zero. However, the results in Fig. 5 may offer a clue to the origin of this coefficient since its magnitude is very nearly constant over several orders of magnitude of change in impurity concentration. This rules out a change in the relaxation time with stress as the origin of π_{44} since the relaxation time in the unstressed state changes by over an order of magnitude, and the mechanism which limits the relaxation time changes over this impurity concentration range. Since a [111] stress is symmetrical to all of the conduction band minima, no electron transfer between minima will occur. The absence of an impurity-concentration dependence in π_{44} also rules out changes in the carrier concentration due to the influence of the stress on the ionization energy of the impurities since the ionization energy goes to zero at the higher impurity concentrations. It is therefore postulated that the π_{44} coefficient may be due to a decrease in the transverse effective mass with a [111] directed tensile stress since a stress-induced effective-mass change could be independent of impurity concentration. This cannot be verified, how-

[45] H. J. McSkimin, J. Appl. Phys. 24, 988 (1953).

[46] H. Fritzsche, Phys. Rev. 115, 336 (1959).

[47] K. J. Schmidt-Tiedemann, in *Proceedings of the International Conference on Semiconductor Physics, Exeter, 1962* (The Institute of Physics and The Physical Society, London, 1962), p. 191.

ever, on the basis of the present piezoresistance experiments.

The sharp increase of π_{44} for samples doped to approximately 1×10^{20} cm^{-3} would seem to indicate the approach of the Fermi level to other energy minima in the conduction band. However, due to the large density-of-states effective mass in n-type silicon, the Fermi level is only approximately 0.08 eV above the bottom of the lowest lying minima in the most heavily doped sample and it is not likely that another set of minima are this close to the lowest lying minima. A possible explanation of the π_{44} behavior may be seen by examining the position of the lowest lying minima with respect to the edge of the Brillouin zone. The lowest minima are located along the [100] reciprocal axes at a wave-vector value equal to[48] 0.86 of the maximum wave vector in the [100] direction as shown in Fig. 10. If the curvature of the band is assumed to be given by the longitudinal effective mass, the energy of the zone boundary is only 0.11 eV above the bottom of the band minima. Two bands are degenerate at the [100] zone boundary, so a distortion in the energy surfaces would be expected to occur in the region of the boundary.[49] Also, the application of a stress along a [111] axis will split this degeneracy and cause a change in the energy surfaces near the boundary. Since the Fermi level in the most heavily doped sample is within approximately 0.03 eV of the energy of the zone boundary, it is felt that effects due to the degeneracy at the zone boundary could cause the observed increase in π_{44}. However, a detailed analysis of the nature of these effects cannot be made from the present results.

VI. CONCLUSION

The piezoresistance effect has been investigated in n-type silicon having impurity concentrations from approximately 1×10^{15} to 1×10^{20} cm^{-3}. From the analysis of the piezoresistance results, it is concluded that for Fermi energies up to 0.08 eV above the band edge, the total number of states in the conduction band can be very closely approximated by assuming the band to be parabolic with a density-of-states effective mass equal to that found in pure silicon.

The piezoresistance effect in n-type silicon is sensitive to the value of K, the mobility anisotropy. However, a quantitative evaluation of K as a function of impurity concentration and temperature requires a knowledge of j, the energy dependence of the relaxation time, which cannot be determined from the low-stress piezoresistance results. On the basis of other experimental data, a value of j of -0.5 is assumed and, with this assumption, K is found to decrease to about 2.5 in the region of moderate doping ($\sim10^{18}$ cm^{-3}) and increase again to approximately 5 at the highest impurity concentrations. Due to

[48] G. Feher, Phys. Rev. **114**, 1219 (1959).

[49] F. Herman, Rev. Mod. Phys. **30**, 102 (1958); L. Kleinman and J. C. Phillips, Phys. Rev. **118**, 1153 (1960).

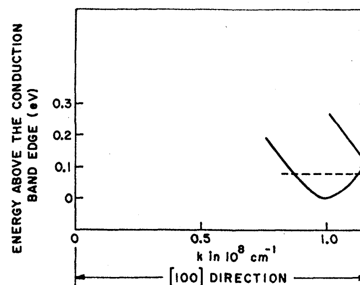

FIG. 10. The detailed shape of the energy bands in n-type silicon in the [100] direction near the band edge. The curvature of the band has been assumed to be given by the longitudinal effective mass. The dashed line shows the position of the Fermi level in the most heavily doped sample (1.5×10^{20} cm^{-3}).

the assumption involved, the K values deduced from the present results must be considered very qualitative.

The temperature dependence of the π_{11} coefficient is found to be anomalous with respect to the predictions of the electron-transfer mechanism and the anomaly is found to be present over the entire range of concentrations. This behavior is explained by assuming the deformation potential increases with temperature at temperatures below approximately 250°K.

Finally, the magnitude of the π_{44} coefficient is found to be independent of impurity concentration for concentrations up to approximately 5×10^{19} cm^{-3} and then increases sharply at higher concentrations. Since the Fermi energy in the most heavily doped samples is only 0.08 eV above the bottom of the conduction band, this behavior is attributed to the Fermi level approaching the energy of the conduction band at the [100] zone boundary and a distortion of the energy bands at this point, rather than the presence of other conduction-band minima.

ACKNOWLEDGMENTS

The authors wish to acknowledge the contribution of Dr. Donald Long and Dr. J. D. Zook through helpful and stimulating discussions during the course of this work.

APPENDIX. CALCULATION OF THE RELATION BETWEEN π_{11} AND n FOR A NONPARABOLIC CONDUCTION BAND

Suppose the conduction band is nonparabolic such that both the longitudinal and transverse effective masses increase with energy as $m^*=m_0^*(1+\alpha E)$, where m_0^* is the effective mass at the band edge. In the limit of $\alpha E\ll1$, the density-of-states function, defined by $g(E)=(k/\pi)^2dk/dE$, may then be written as

$$g(E)=4\pi\left(\frac{2m_{od}^*}{h^2}\right)^{3/2}E^{1/2}(1+\tfrac{5}{2}\alpha E)\,, \qquad \text{(A1)}$$

where m_{0d}^* is the density-of-states effective mass at the band edge. If this density-of-states function and a

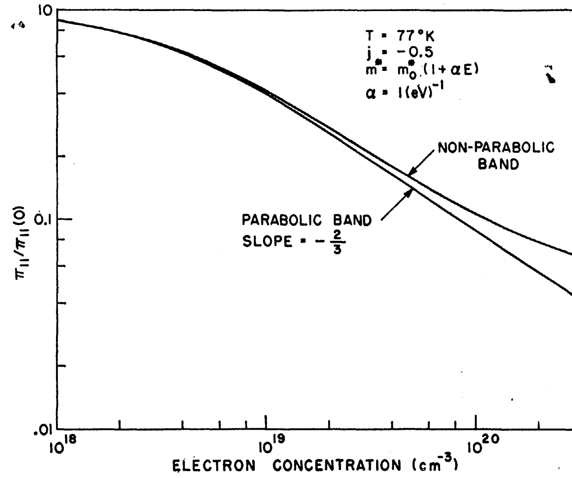

FIG. 11. The calculated reduction in π_{11} due to statistical degeneracy for a parabolic conduction band with a density-of-states effective mass of 1.06 m_0 and a nonparabolic band with the same density of states at the band edge.

relaxation time of $\tau=\tau_0E^j$ are used to calculate the change in the conductivity tensor of a single valley due to a change δE in the energy of the valley, one obtains for each component

$$\frac{\delta\sigma}{\sigma}=\frac{1}{kT}\left[\frac{\mathfrak{F}_{j-1/2}(\eta)+\frac{3}{2}\alpha(j+\frac{5}{2})kT\mathfrak{F}_{j+1/2}(\eta)}{\mathfrak{F}_{j+1/2}(\eta)+\frac{3}{2}\alpha(j+\frac{5}{2})kT\mathfrak{F}_{j+3/2}(\eta)}\right]\delta E. \quad \text{(A2)}$$

The coefficient π_{11} for n-type silicon may then be written as

$$\pi_{11}=\pi_{11}(0)\left[\frac{\mathfrak{F}_{j-1/2}(\eta)+\frac{3}{2}\alpha(j+\frac{5}{2})kT\mathfrak{F}_{j+1/2}(\eta)}{\mathfrak{F}_{j+1/2}(\eta)+\frac{3}{2}\alpha(j+\frac{5}{2})kT\mathfrak{F}_{j+3/2}(\eta)}\right]. \quad \text{(A3)}$$

Expanding the Fermi-Dirac functions by Eq. (6), one obtains for the limit of large η

$$\pi_{11}=\pi_{11}(0)\frac{j+\frac{3}{2}}{\eta}\left[\frac{1+\frac{3}{2}\alpha[(j+\frac{5}{2})/(j+\frac{3}{2})]kT\eta}{1+\frac{3}{2}\alpha kT\eta}\right]. \quad \text{(A4)}$$

The value of the bracket term in Eq. (A4), which results from the nonparabolic nature of the band, depends on the value of α and j which are not usually known. However, if $j>-1$, which will be true in most of the physically realizable situations, the value of the bracket term will be greater than unity so that the effect of a nonparabolic band will be to make π_{11} larger for a given value of η.

The relation between n and η for the density-of-states function given by Eq. (A1) is given by

$$n=4\pi\left(\frac{2m_{od}{}^*kT}{h^2}\right)^{3/2}[\Gamma(\tfrac{3}{2})\mathfrak{F}_{1/2}(\eta)+\tfrac{5}{2}\alpha kT\Gamma(\tfrac{5}{2})\mathfrak{F}_{3/2}(\eta)], \quad \text{(A5)}$$

which when expanded for the case of large η gives

$$n=\frac{8\pi}{3}\left(\frac{2m_{od}{}^*kT}{h^2}\right)^{3/2}\eta^{3/2}[1+\tfrac{3}{2}\alpha kT\eta]. \quad \text{(A6)}$$

This gives the expected result that the nonparabolic nature of the band will make η less for a given n since the density of states is increasing with energy faster than in the parabolic case. This will also have the effect of making π_{11} larger for a given n in the nonparabolic case. Thus, the change in the relation of π_{11} to η and of n to η due to the nonparabolicity of the band will both tend to make π_{11} decrease less rapidly with increasing n.

In order to estimate the sensitivity of the slope of a plot of the ln π_{11} versus ln n to nonparabolic effects, the predictions of Eqs. (A3) and (A5) have been compared with the predictions for a parabolic band given by Eqs. (7) and (8). For the purposes of comparison, it has been assumed that $\alpha=1$ eV^{-1}, and that $j=-0.5$ and $T=77$°K. The results are shown in Fig. 11. For n-type silicon, the Fermi energy calculated by assuming a parabolic band and a density-of-states effective mass of 1.06 m_0 is approximately 0.07 eV for an impurity concentration of 1×10^{20} cm^{-3}. Therefore, the change in m^*, which is given approximately by αE_f, is 7% at an impurity concentration on 1×10^{20} cm^{-3}. However, this comparatively small change causes a 14% difference in the π_{11} values in Fig. 11 and produces a slope that would be detectably different from the $-\frac{2}{3}$ value predicted for a parabolic band. If j is closer to zero or positive, the discrepancy between the two curves in Fig. 11 will be smaller, but a detectable difference will still be present. It is therefore concluded that the piezoresistance effect should be a sensitive method of investigating the parabolicity of the conduction band in silicon.

In the preceding discussion, it was assumed that the effective mass increases with energy, i.e., α is positive. If α is negative, such that m^* decreases, it is clear that the slope of a logarithmic plot of π_{11} versus n will be greater than $-\frac{2}{3}$. If the band were to have a variable curvature such that α is both positive and negative at different energies, the net result could appear approximately as a parabolic band on the basis of the piezoresistance results. It was also assumed that both the longitudinal and transverse effective masses increase with energy in the same manner such that the ratio of $m_{\parallel}/m_{\perp}$ remains unchanged. Actually, nonparabolic effects may show up at a lower energy in $m_{\perp}$ than $m_{\parallel}$. In this case, the mobility anisotropy K will also depend on energy so that a change in $\pi_{11}(0)$ in addition to a change due to the previously calculated density-of-states effect will be present. However, a quantitative calculation of the change in K, due to a change in $m_{\parallel}/m_{\perp}$ cannot be made without a detailed understanding of the scattering mechanisms in heavily doped silicon. Qualitatively, the change in $\pi_{11}(0)$ will tend to decrease the change in π_{11} due to the density-of-states effect.